(S) 508400
(7) NA
(8) NA
(9) NA
(11) Dec 62
(12) [15]p.
(13-14) NA

(18)   (19)
AFCRL 63-218

(6) MANTLE SHEAR WAVE VELOCITIES DETERMINED FROM OCEANIC
LOVE AND RAYLEIGH WAVE DISPERSION

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(15) Contract AF19 (604) 7376

(16) Project No. 8652

(17) Task No. 86522

(20) U
(21) In cooperation with Tokyo U. (Japan)

December 1962

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Mantle Shear Wave Velocities Determined from Oceanic Love and Rayleigh Wave Dispersion[1]

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Abstract. Dispersion of Love and Rayleigh waves has been computed for a model of the suboceanic crust and mantle. An important feature of this model is a low-velocity layer in the upper mantle. These calculations, which include the effect of sphericity, are consistent with oceanic Love wave dispersion data for periods from 10 and 800 seconds to within a few hundredths of a kilometer per second. The effect of sphericity on oceanic Love waves is of great importance for periods as short as 10 seconds because of the penetration of these waves into the low-velocity channel. Computations for the same model, which consists of homogeneous isotropic layers, are also in accord with the oceanic Rayleigh wave data for periods from 30 to 150 seconds to within a few hundredths of a kilometer per second. The results can be further improved by minor modifications of the model. Since the calculations for a spherical earth are consistent with oceanic Love and Rayleigh wave data, apparent discrepancies which result from computations for flat layers are resolved without recourse to a difference in *SH* and *SV* velocities in the suboceanic mantle. Recent calculations and observations of Love and Rayleigh wave dispersion by Brune and Dorman for the mantle beneath the Canadian shield showed (1) that a low-velocity zone is required and (2) that no difference between *SH* and *SV* velocities is needed. Thus their study and this paper show that these two conclusions are applicable to large areas of the world. The wave guide of the *G* wave for periods from 60 to 300 seconds is the free surface and the steep gradient in shear velocity in the upper half of the mantle. At shorter periods the fundamental oceanic Love mode exhibits large particle amplitudes in the low-velocity channel. A comparison of the upper mantle beneath oceans and continents confirms previous results which indicate that channel velocities are smaller beneath the oceans.

Introduction. The physical properties of the crust and mantle beneath oceanic areas can be investigated with the aid of Love and Rayleigh waves. In this study theoretical phase and group velocities for a model which contains a low-velocity layer are compared with experimental data for a broad range of periods. The effect of sphericity on Love and Rayleigh waves has been taken into account in the theoretical calculations.

In this paper considerable emphasis has been placed on the Love wave solution for a spherical earth as compared with the result for flat layers. The effect of sphericity on the phase and group velocities of oceanic Love waves is not negligible, even for periods as short

as 10 seconds, because of the penetration of the wave disturbance into the low-velocity channel. In fact, sphericity can resolve the apparent discrepancies discussed by *Ewing and Landisman* [1961] which result from computations of Love and Rayleigh wave dispersion for flat layers. When sphericity is taken into account there is no longer a necessity for a difference in *SH* and *SV* velocities in the suboceanic mantle.

Recently *Brune and Dorman* [1962], using high-precision Rayleigh and Love wave data, found that no difference between *SH* and *SV* velocities is needed for the mantle beneath the Canadian shield. They also demonstrated that a low-velocity layer is required for this region. Thus their study and this paper show that these two conclusions are applicable to large areas of the world.

For the calculations used in this study, the

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[1] Lamont Geological Observatory Contribution 578.

distribution of velocities and densities for the suboceanic crust and mantle was selected with the aid of several different criteria. The physical parameters for the sedimentary and crustal layers of case 122 were chosen to be in accord with the results of seismic refraction work at sea and with the short-period oceanic surface wave results of *Oliver and Dorman* [1961]. For the mantle the distribution of physical parameters for case 122, which is consistent with data for teleseismic body waves, is the same as that for oceanic case 8099 of *Dorman et al.* [1960]. These authors showed that case 8099 was in accord with the group velocity data for mantle Rayleigh waves determined by *Ewing and Press* [1954a, b] and with the oceanic Rayleigh wave observations of *Sutton et al.* [1959]. One of the most important results demonstrated by Dorman et al. was the requirement of a low-velocity channel for the mantle beneath continents and oceans.

Recent observational work has produced additional Rayleigh wave data for the oceans, some of which includes phase velocities as well as group velocities [*Aki*, 1960a, b; *Aki and Press*, 1961; *Kuo et al.*, 1962]. Kuo et al. have shown that case 8099 explains the major features of oceanic mantle Rayleigh wave dispersion. The present study indicates that theoretical Love and Rayleigh wave phase and group velocities computed for case 122 satisfy available oceanic dispersion data.

Long-period Love waves, especially G waves, have been studied since 1926 by Gutenberg, Byerly, Imamura, Wilson, Satô, and others. Love wave calculations by *Satô* [1958], *Landisman and Satô* [1958], and *Landisman et al.* [1959] have contributed to the present knowledge of the upper mantle. With the exception of those of *Satô* [1958], these Love wave calculations, studies of teleseismic body wave times and amplitudes, and the Rayleigh wave computations cited above have shown that beneath the crust the upper mantle has high-velocity material which overlies a region of lower velocities. At even greater depths higher velocities are also required. This distribution of velocities is compatible with those inferred from body wave studies by *Gutenberg* [1953] and *Lehmann* [1955]. A general conclusion to be drawn from these surface wave investigations is that the channel velocities in the upper mantle are

smaller beneath the oceans than beneath the continents.

The first studies in which observational group velocity data were compared with calculations for flat layers were restricted either to Love waves or to Rayleigh waves. Most of the resulting independent velocity distributions confirmed the existence of the low-velocity layer, but they were quite dissimilar in detail. For example, *Dorman et al.* [1960] demonstrated that case 38 km-XII, which had been shown by *Landisman and Satô* [1958] to explain continental Love wave data, was not in accord with continental Rayleigh wave data. *Ewing and Landisman* [1961] discussed this problem further, paying particular attention to the mantle structure beneath oceans, and presented a figure showing the oceanic and continental shear velocity distributions derived from calculations for flat layers. These authors emphasized that a proper treatment of the problem must include sphericity.

Theoretical studies have recently been made which include allowance for sphericity, gravity, gradients, and a liquid core. The spheroidal oscillations have been treated theoretically by *Alterman et al.* [1961], *Bolt and Dorman* [1961], and others. The torsional oscillations have been investigated by *Jobert* [1959, 1960a, b], *Satô et al.* [1960a, b], *Pekeris et al.* [1961], *MacDonald and Ness* [1961], and others. *Jobert* [1960b] presented Love wave phase and group velocity curves for a modified Gutenberg mantle which was considered appropriate for oceanic areas. This calculation for a spherical earth produced a fairly flat group velocity curve with a maximum of approximately 4.5 km/sec at 30 seconds and a minimum of approximately 4.4 km/sec near 3 minutes. Although no comparison was made with observed data, the maximum and the minimum lie slightly outside the range of observed group velocities for the fundamental oceanic Love mode.

Some of the theoretical studies cited above were stimulated by the occurrence of the great Chilean earthquake of May 22, 1960, which excited the free vibrations reported by *Alsop et al.* [1961], *Benioff et al.* [1961], *Bogert* [1961], *Ness et al.* [1961], *Brune, Benioff, and Ewing* [1961], and others. These studies of the Chilean earthquake produced new and precise determinations of the dispersion of long-period

Love and Rayleigh waves. For periods greater than several hundred seconds, the steep rise in phase and group velocities, which had been predicted by theory, was confirmed by observations. This steep rise is caused by the penetration of the wave disturbance into the lower half of the mantle.

Computational procedures. Love wave dispersion for a spherical earth, consisting of a large number of spherical shells, has been computed using an IBM 7090 version of a numerical iteration program described by *Satô et al.* [1960a, b]. This program has been used to calculate free periods of torsional oscillation, phase velocity, group velocity, particle amplitude, and strain distribution as a function of depth for each of the azimuthal mode numbers *n*. Group velocity, which is calculated by the polynomial method, is about two orders of magnitude more precise than the observational data. With this program dispersion can be computed for a spherical or a flat-layered model; the same physical parameters and numerical procedures can be used in both cases. These calculations for flat layers were compared with those made with the aid of the Thomson-Haskell [*Haskell*, 1953] matrix program, PV7, written by Dorman [*Oliver and Dorman*, 1961], and the results agree at all periods to better than 0.0001 km/sec. In addition, the matrix method was combined with the earth-flattening approximation [*Alterman et al.*, 1961] as a further test of the calculations for the spherical model. All methods give very similar fundamental Love mode phase velocities for periods shorter than 8 seconds. For periods shorter than 50 seconds the earth-flattening approximation agrees to better than 0.01 km/sec with the fundamental Love mode phase and group velocities determined by the calculations for a spherical earth.

A program written by L. E. Alsop was used in computing oceanic Rayleigh wave free periods, phase and group velocities, particle amplitudes, and gravitational disturbance for each azimuthal mode of a gravitating, spherical earth with a liquid surface layer. This routine is similar to one described by *Alsop* [1962]. The oceanic Rayleigh mode phase and group velocities which resulted were then used to check the correction for sphericity for *continental* Rayleigh waves reported by *Bolt and Dorman*

[1961], which was applied to Dorman's *oceanic* case 8099 by *Kuo et al.* [1962].

For periods between 30 and 150 seconds, the phase and group velocities calculated for case 122 agree with the theoretical values for case 8099S (spherical) of Kuo et al. to within 0.01 km/sec. Rayleigh wave calculations for a flat-layered version of case 122 also agree with Dorman's published values for case 8099 to within 0.01 km/sec over the same range of periods. For periods shorter than 50 seconds the earth-flattening approximation for case 122 agrees to better than 0.01 km/sec with the Rayleigh mode phase and group velocities for a spherical earth.

A matrix program for liquid and solid flat layers was used in computing Rayleigh wave particle amplitudes. The method is similar to that reported by *Dorman and Prentiss* [1960] and *Dorman* [1962].

Oceanic case 122 compared with other studies of the crust and mantle. For the computations of spherical Love and Rayleigh waves for the model reported in this study, the velocities and densities for the crust and mantle were chosen to be in accord with recent geophysical studies of oceanic areas. The velocities for the sedimentary and crustal layers of case 122 were indicated by the recent work of *Oliver and Dorman* [1961] and are consistent with seismic refraction work at sea as reported by *Raitt* [1956] and *Ewing and Ewing* [1959]. Crustal densities adopted for case 122 conform to the Nafe and Drake empirical velocity-density curve reported by *Talwani et al.* [1959]. The layer parameters for the homogeneous isotropic layers which constitute case 122 are given in Table 1. For the crust, these layers correspond closely in both thickness and velocity to those indicated by seismic refraction. *Ewing and Ewing* [1961], using a seismograph on the ocean floor, obtained subcrustal shear velocity measurements at small distances. These authors consider their measurements to be appropriate only for the upper surface of the mantle, and for this reason their value of mantle shear velocity was not used for case 122.

For the mantle the layer parameters for case 122, presented in Table 1, are identical to those for oceanic case 8099 of Dorman et al. The homogeneous layers for the mantle were required by Dorman et al. because of their method of calculation. These layers are a stepped ap-

TABLE 1. Physical Parameters for the Homogeneous, Isotropic, Spherical Shells of Case 122

Thickness (km), compressional velocity α (km/sec), shear velocity β (km/sec), and density ρ (g/cm³). Earth radius = 6370 km and core radius = 3470 km for the calculations for the spherical model. Deepest layer represents the half-space for calculations for the flat-layered model.

LAYER NO.	THICKNESS KM	α KM/S	β KM/S	ρ GM/CC
1	5.0	1.52	0.0000	1.0300
2	0.5	2.00	0.5000	1.9000
3	1.5	4.80	2.7700	2.5400
4	5.0	6.90	3.9800	2.9000
5	49.0	7.82	4.6125	3.3400
6	160.0	8.17	4.3000	3.4425
7	100.0	8.49	4.6000	3.5265
8	90.0	8.81	4.8000	3.6040
9	90.0	9.32	5.1925	3.7650
10	100.0	9.97	5.4925	4.0100
11	100.0	10.48	5.7900	4.2300
12	100.0	10.85	6.0300	4.4700
13	100.0	11.12	6.2000	4.5450
14	100.0	11.33	6.3150	4.6400
15	100.0	11.49	6.4000	4.7100
16	100.0	11.64	6.4650	4.7700
17	100.0	11.78	6.5310	4.8275
18	100.0	11.92	6.5910	4.8825
19	100.0	12.06	6.6490	4.9400
20	100.0	12.19	6.7040	5.0000
21	100.0	12.33	6.7550	5.0550
22	100.0	12.46	6.8050	5.1050
23	100.0	12.59	6.8525	5.1575
24	100.0	12.72	6.8975	5.2075
25	100.0	12.85	6.9450	5.2650
26	100.0	12.97	6.9950	5.3150
27	100.0	13.09	7.0450	5.3650
28	100.0	13.21	7.0950	5.4150
29	100.0	13.33	7.1425	5.4650
30	100.0	13.46	7.1875	5.5150
31	100.0	13.53	7.2325	5.5625
32	100.0	13.61	7.2775	5.6075
33		13.64	7.3000	5.6550

proximation to velocities and densities which are based on those originally determined from earth-quake body wave studies. In the present study, the homogeneous isotropic layers of case 8099 of Dorman et al. have been retained for case 122.

Observations of the travel times of P and S from nuclear explosions in the Pacific by Carder and Bailey [1958], Kogan [1960], and others have confirmed the teleseismic travel time curves remarkably well. For sources in the Pacific and recording stations on continents, P arrives 2 seconds early and S is recorded 4 + seconds late, as compared with the Jeffreys-Bullen tables. Ex-

cept for one observation of S near 17° [Pomeroy, 1962], no travel-time data are available from nuclear explosions in oceanic areas at distances less than 34° for S and 3½° for P. The short-period S wave observation near 17° indicates a minimum upper-mantle shear velocity of 4.7 km/sec beneath the ocean basin between Eniwetok and Guam. No oceanic refraction results exist for distances greater than 1°. These precise results must be further amplified in order that they may give information about the suboceanic upper mantle.

It is instructive to compare the mantle velocities determined from surface wave studies

of oceanic and continental areas. The sub-oceanic mantle distributions to be considered are case 122, case 8099 of *Dorman et al.* [1960], and case 6EGHP1 of *Aki and Press* [1961]. These distributions are very similar; all contain a channel with a minimum shear velocity of 4.30 km/sec. As noted above, cases 122 and 8099 are identical in the mantle. The upper and lower channel boundaries of case 6EGHP1 are not as sharp as those for cases 122 and 8099, since case 6EGHP1 was obtained by a modification of a Gutenberg instead of a Lehmann continental distribution.

For the continents the Gutenberg-Bullen A model, case 6EGH of *Aki and Press* [1961], and the distribution CANSD determined from the dispersion of Love and Rayleigh waves across the Canadian shield [*Brune and Dorman*, 1962] will now be considered. The Gutenberg-Bullen A model has been used to explain the observed mantle Love and Rayleigh wave data from the great Chilean earthquake and the southeast Alaska shock of July 10, 1958 (e.g., *Brune, Benioff, and Ewing* [1961]; *Pekeris et al.* [1961], etc.). Case 6EGH is a model based on one of Gutenberg's later velocity distributions. The principal difference between case 6EGH and the Gutenberg-Bullen A model occurs between 50 and 100 km, where case 6EGH has lower shear velocities. The Canadian shield case, CANSD, is characterized by higher velocities than either of these models. The shear velocity from the Mohorovicic discontinuity to a depth of 115 km is 4.72 km/sec. The low-velocity channel, which is 200-km thick, consists of a 4.54-km/sec layer overlying a 4.51-km/sec layer of equal thickness.

In general, surface wave studies to date have shown that the channel velocity is lower beneath the oceans than beneath the continents. In addition, with the exception of the study of *Aki and Press* [1961], several of these investigations have indicated that the oceanic channel velocities occur nearer the surface. Below the M discontinuity, the physical parameters for the continental model 6EGH and the oceanic model 6EGHP1 of Aki and Press differ only in the value of shear velocity chosen for the low-velocity zone. Model 6EGH was compared with only one seismogram for periods near the continental Airy-phase maximum. Further study and a comparison of various models with more

continental data are needed in order to determine whether channel velocities usually occur at greater depths beneath the continents than beneath the oceans.

Love wave phase and group velocities. Experimental and theoretical fundamental mode phase velocities for oceanic Love waves of periods up to 800 seconds are presented in Figure 1. Experimental data appropriate for oceanic areas have been taken from *Satô* [1958], as corrected for the polar phase shift [*Brune, Nafe, and Alsop*, 1961; *Brune, Benioff, and Ewing*, 1961; *Benioff et al.*, 1961; *MacDonald and Ness*, 1961]. Of these, Satô's data have the greatest percentage of oceanic path and hence his data are considered more reliable. Four theoretical curves are shown; one is the result of a calculation for flat layers, and the other three are for a spherical earth. Numerical results of the calculations of Love waves for the spherical version of case 122 appear in Table 2. For azimuthal mode numbers less than 360, the second and third layers (shear velocity $\beta = 0.5$ and 2.77 km/sec, respectively) were deleted and replaced by a single 2-km layer with a shear velocity of 2.77 km/sec and a density of 2.54 g/cm³. A detailed comparison of these computations with data will be made in conjunction with the material presented in Figure 2.

Calculated group velocities for two flat-layered and two spherical models are compared with group velocity data appropriate for oceanic areas in Figure 2. The sources of these data are indicated in the figure. In the long-period Chile to Ñaña data, those represented by stars are the more reliable, according to *Brune, Benioff, and Ewing* [1961], because the effects of initial phase at the source have been removed. The symbol G represents values which were picked from the summary curve of Love wave data for the Pacific in Figure 1 in the third paper of *Gutenberg and Richter* [1936, p. 89]. The data indicated by squares are from a southeast Pacific shock which occurred on November 14, 1958, at 05h 04m 25s GCT at 36.0°S, 102.8°W, as recorded at Hallett station, Antarctica.

Several observations may be made from a study of Figures 1 and 2. The calculation of case 122 for flat layers departs significantly from the Love wave phase and group velocity data in Figures 1 and 2. In contrast, when the computation is done for a spherical earth, the phase
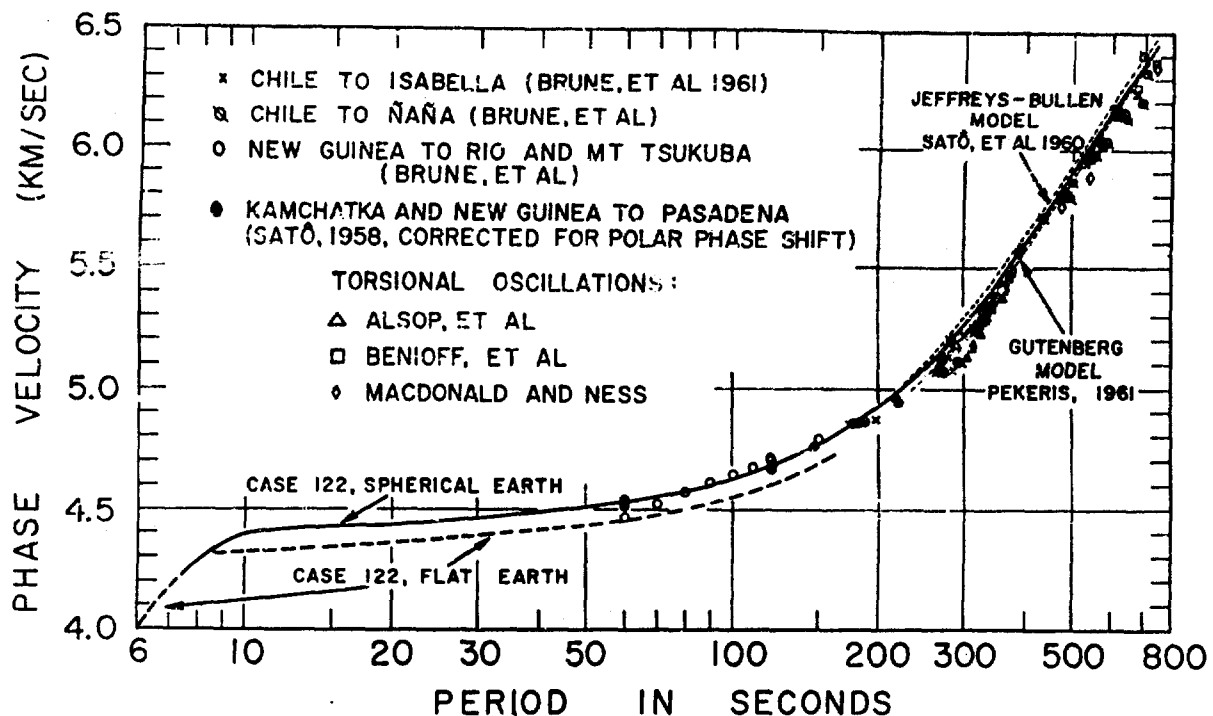
Fig. 1. Phase velocities for fundamental-mode oceanic Love waves. Calculation for a spherical earth (solid line) agrees more closely with observational data than calculation for a flat-layered model (dashed line). Parameters for case 122 in Table 1.

velocities agree with available data for periods between 60 and 800 seconds to within a few hundredths of a kilometer per second. The corresponding group velocities compare well with existing data over an even broader range of periods. The agreement of theoretical wave velocities for case 122 with observational data over such a broad range of periods indicates that the physical parameters chosen are close to the actual ones in the earth to depths of at least several hundred kilometers beneath the ocean. These results may be further improved by minor modifications of the model. New and more precise data are also needed.

If no calculations for a spherical earth were available, one might tend to modify the physical parameters of case 122 in order to explain the Love wave phase and group velocity data. One of the better Love wave calculations for flat layers is the case marked Oceans VIII [Landisman et al., 1959]. The shear velocity distribution for this case was presented by Ewing and Landisman [1961]. The calculation of phase and group velocities for this model was the first quantitative attempt to explain the dispersion of oceanic Love waves with a mathematical model containing a low-velocity channel. This

model indicated that oceanic channel velocities are smaller and come nearer to the surface than continental channel velocities. As was discussed above, several recent studies corroborate this result. In comparison with cases 122 and 8099, the shear velocity for Oceans VIII is higher by several tenths of a kilometer per second in the lower part of the channel. The differences between the shear velocity distributions for Oceans VIII, derived from Love wave dispersion, and case 8099 of Dorman et al., determined from Rayleigh wave data, were one of the discrepancies noted by Ewing and Landisman. Similar discrepancies between Love and Ralyeigh wave results have been the basis of recent speculations concerning anisotropy in the upper mantle [Anderson, 1961, 1962; Anderson and Harkrider, 1962]. Since case 122 agrees with the data when calculation for a spherical earth is performed, no recourse to a difference in SH and SV velocities, i.e., anistropy, is required. Thus one distribution of homogeneous isotropic layers can satisfy both Love and Rayleigh wave data.

In Figures 1 and 2, the higher velocities at longer periods have been measured using data from the great Chilean earthquake. The theoreti-

TABLE 2. Love Wave Periods T (sec), Phase Velocities C (km/sec), and Group Velocities U (km/sec) for Various Azimuthal Mode Numbers N Computed with the Spherical Earth Program for Case 122

For N less than 360 the second and third layers ($\beta = 0.5$ and 2.77) were deleted and replaced by a single 2-km layer having $\beta = 2.77$, $\rho = 2.54$.

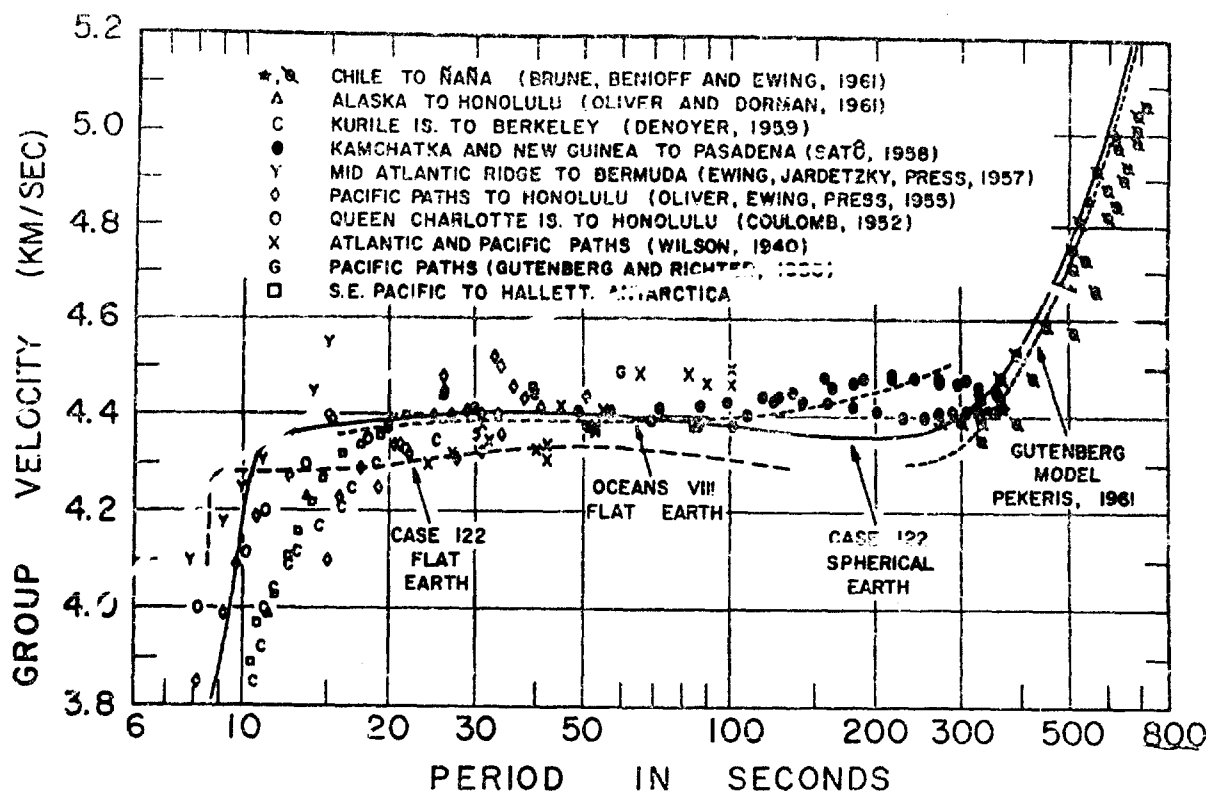| N	T SEC	C KM/S	U KM/S	N	T SEC	C KM/S	U KM/S
7	812.79	6.5657		96	90.054	4.6056	4.3844
8	731.52	6.4368		108	80.518	4.5814	4.3689
9	667.21	6.3144	5.1911	126	69.470	4.5544	4.3941
10	614.74	6.2007	5.0531	144	61.081	4.5347	4.3981
11	570.94	6.0958	4.9405	175	50.556	4.5210	4.4026
12	533.69	5.9996	4.8474	225	39.553	4.4874	4.4058
13	501.54	5.9113	4.7696	245	36.385	4.4808	4.4060
14	473.44	5.8302	4.7044	285	31.361	4.4702	4.4050
15	448.63	5.7558	4.6495	360	24.922	4.4549	4.3993
16	426.57	5.6873	4.6033	380	23.627	4.4519	4.3976
17	406.65	5.6242	4.5642	400	22.462	4.4492	4.3957
18	388.69	5.5660	4.5312	420	21.406	4.4466	4.3938
19	372.36	5.5122	4.5033	440	20.445	4.4441	4.3919
20	357.42	5.4624	4.4797	460	19.567	4.4418	4.3900
21	343.71	5.4162	4.4597	480	18.762	4.4396	4.3880
22	331.05	5.3733	4.4428	500	18.021	4.4375	4.3861
23	319.34	5.3334	4.4284	520	17.336	4.4355	4.3842
24	308.45	5.2962	4.4163	540	16.702	4.4336	4.3823
25	298.31	5.2615	4.4063	560	16.113	4.4317	4.3805
26	288.84	5.2290	4.3973	638	14.166	4.4250	4.3733
27	279.96	5.1987	4.3899	683	13.244	4.4214	4.3688
28	271.63	5.1702	4.3837	728	12.435	4.4180	4.3633
29	263.78	5.1434	4.3785	818	11.086	4.4109	4.3380
30	256.39	5.1183	4.3741	863	10.519	4.4062	4.2949
33	236.53	5.0512	4.3648	908	10.016	4.3983	4.1825
36	219.55	4.9945	4.3597	953	9.574	4.3844	4.0314
39	204.86	4.9462	4.3573	1013	9.059	4.3592	3.8965
45	180.68	4.8685	4.3570	1058	8.717	4.3379	3.8213
51	161.61	4.8090	4.3595	1103	8.405	4.3154	3.7523
60	139.49	4.7425	4.3650	1163	8.030	4.2841	3.6628
72	117.94	4.6807	4.3725	1208	7.775	4.2597	
84	102.14	4.6374	4.3790	1268	7.466	4.2261	3.5956

Fig. 2. Group velocities for fundamental-mode oceanic Love waves. Calculation for spherical earth case 122, compared with observational data from 10 to 800 seconds period. For discussion see text.

cal curves for the Jeffreys-Bullen and Guten-berg-Bullen A continental models shown in Figures 1 and 2 agree quite well with the results of case 122 at long periods.

For periods shorter than 12 seconds the lower-velocity oceanic crustal materials, in particular the sediments, produce a sharp drop in group velocity as the period decreases. For periods between 10 and approximately 50 seconds, small segments of continental path can lower the observed group velocities. For the data from Hallett station, a correction was applied for the 6 per cent of the path that lies within the continent of Antarctica.

The wave guide of the G wave. The well-known pulselike character of the G wave, which is best observed for oceanic paths, is produced by the nearly constant group velocity of the fundamental oceanic Love mode for periods as great as 300 seconds, as shown in Figure 2. In his study of the wave guide of the G wave, Satô [1958] said that 'this phase corresponds to a Love wave traveling in the mantle and controlled by the gradient in shear velocity' between

depths of 100 and several hundred kilometers.

We may further study the wave guide of the G wave with the aid of the Love wave particle amplitudes computed for a spherical earth (Figure 3). For reference, the distribution of shear velocity with depth is shown at the left. The particle amplitudes are normalized to unity at the free surface. The particle amplitude for $T = 59.07$ seconds is representative of particle amplitudes for the G wave for periods between 60 and 300 seconds. For these periods the free surface and the steep gradient in shear velocity down to approximately 900 km form the wave guide of the G wave. For periods greater than 300 seconds, the particle motion penetrates the high-speed lower mantle, and observed Love wave phase and group velocities increase sharply, as had been predicted by theory. Similarly, for periods greater than 250 seconds, observed and theoretical Rayleigh wave phase and group velocities rise even more abruptly than those for Love waves. For these wavelengths, Rayleigh waves penetrate more deeply into the earth.

In Figures 1 and 2, the phase and group ve-
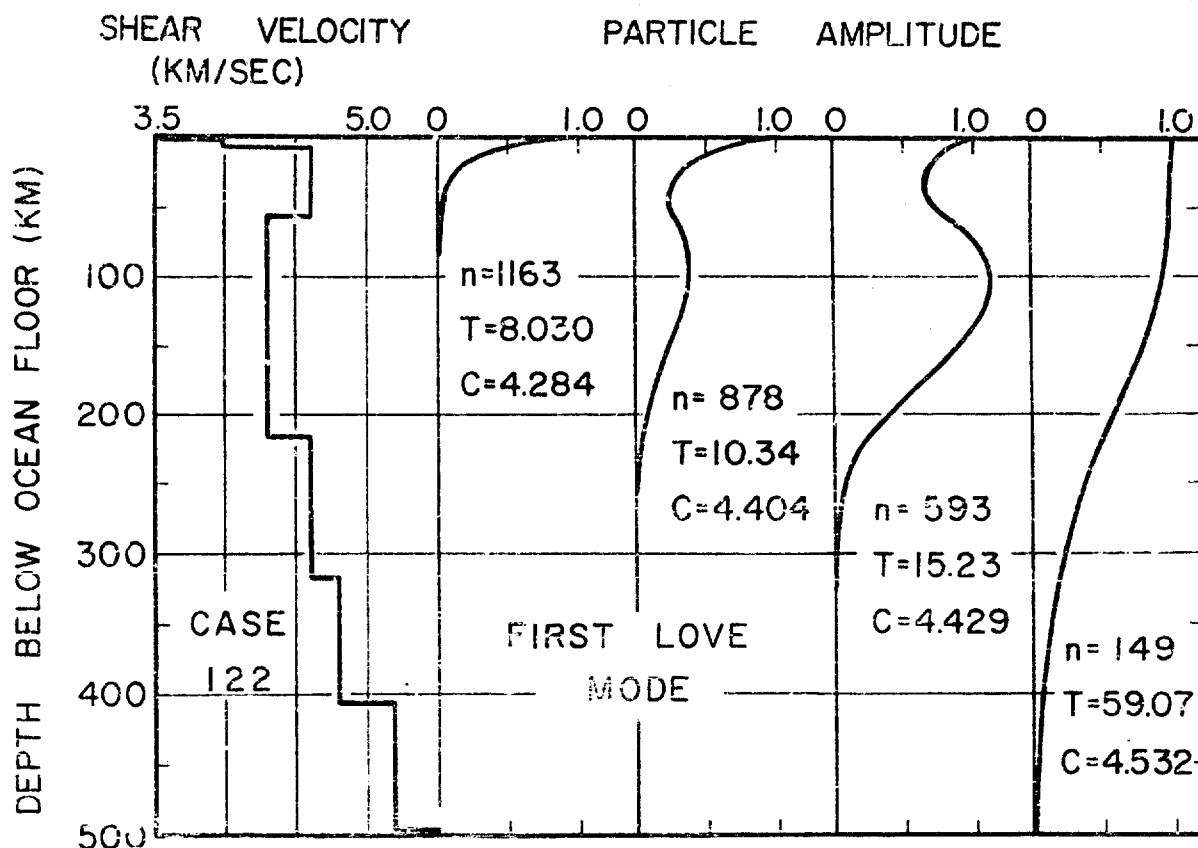
SHEAR VELOCITY PARTICLE AMPLITUDE

Fig. 3. Particle amplitude profiles as a function of depth for the fundamental Love mode of case 122, calculation for spherical model. Amplitudes normalized to unity at the free surface. Secondary maximum develops when phase velocity C begins to exceed 4.3 km/sec, the channel velocity.

locities for case 122, as calculated for a spherical earth and for flat layers, remain approximately parallel for periods between 12 and 150 seconds. For periods shorter than 10 seconds, the two phase velocity curves converge; for periods shorter than 8 seconds, they are indistinguishable. As discussed in the section on computational procedures, both numerical iteration and the matrix method were used for all these calculations and were found to be in good agreement.

Within the period range where the phase and group velocity curves are parallel, the difference between the calculations for the spherical earth and for the flat-layered versions of case 122 is approximately 0.8 km/sec. In simplest terms, this increase in velocity for the spherical earth can be attributed to a shorter path. For the spherical earth, if we imagine that the waves travel predominantly in the low-velocity channel instead of along the surface, the path is

shortened by a factor approximately equal to the ratio of the effective channel depth to the earth's radius (M. Ewing, personal communication). Thus the velocity of the waves is increased by approximately $(z/a) \cdot V_s$, where z is the effective depth to the channel, a the radius of the earth, and V_s the velocity in the channel. In the present case, with a channel velocity of 4.30 km/sec, the difference of 0.08 km/sec in Figures 1 and 2 indicates an effective channel depth of 120 km. This depth lies well within the channel for case 122. With this difference of path length in mind, it is easy to understand how a spherical model with a channel velocity of 4.3 km/sec develops the observed G wave velocity of approximately 4.4 km/sec. Press and Ewing [1956] and Press [1959] have suggested, without detailed calculation, that the comparison of the 4.7-km/sec mantle shear velocity, derived from seismic refraction measurements, with the 4.4-km/sec G-wave velocity indicates

the presence of a low-velocity channel in the upper mantle.

To understand the reasons for the abrupt divergence of the phase velocity curves for spherical and flat models near 10 seconds, we may consider the diagrams of Love wave particle amplitude for a spherical earth shown in Figure 3. At the shortest period, 8.03 seconds, the motion is confined to the crust and to the high-speed layer just below the crust. In this part of the spectrum, the calculations for spherical and flat models give nearly identical phase and group velocities and particle amplitudes. When the period becomes greater than 8 seconds, the phase velocity increases and the waves penetrate farther into the high-speed portion of the mantle. At the period where the phase velocity becomes equal to the channel velocity, drastic changes in particle amplitude begin to occur. Within a period range of approximately 1 second, the wave which was confined above the channel emerges as a disturbance trapped in the entire low-velocity zone. This penetration of the surface waves through the high-speed mantle into the low-velocity channel is analogous to many problems of barrier penetration in quantum mechanics.

Near 10 seconds, where the fundamental oceanic Love mode penetrates the channel, the particle amplitudes for the spherical and flat-layered versions of model 122 differ and the phase velocity curves diverge. Calculations of particle amplitudes for the flat-layered version show more severe trapping. The lower phase velocity and larger channel amplitudes for this case may be attributed to a more effective low-velocity channel.

The particle amplitude curves for the spherical earth version of case 122 at $T = 10.34$ and 15.23 seconds in Figure 3 illustrate trapping and also exhibit secondary maximums. For case 122, the largest secondary maximum occurs near 15 seconds when the calculation is performed for a spherical earth. At these periods, the phase and group velocities and the particle amplitudes which result from the earth-flattening approximation are extremely similar to those calculated for the spherical earth. Thus, for the spherical earth, the simplified explanation discussed previously for the relation between the channel velocity and the G wave velocity is justified by the study of particle amplitude trapping and

secondary maximums in the low-velocity channel for case 122.

There are two requirements for the development of secondary maximums in Love wave particle amplitudes. First, the phase velocity must be slightly higher than the channel velocity but less than the velocity of the higher-speed regions adjacent to the channel. Second, the vertical wavelength must be less than or comparable to the channel thickness.

In a study of oceanic Love waves, Jobert [1960b] showed that, for $n = 600$, the fundamental mode particle amplitude develops a secondary maximum. Satô et al. [1960b] also observed that, for models with a low-velocity layer, the higher-mode particle amplitudes can be much greater in the channel than at the surface.

It is evident that secondary maximums will play an important role in the problem of normal mode excitation by earthquakes at depth. Caloi [1953, 1954] and Gutenberg [1954, 1955] reported that earthquakes within the low-velocity channel tend to excite the phases P_* and S_* quite easily. Bolt and Dorman [1961] calculated the group velocity of the first shear mode for a Gutenberg velocity distribution. They found a maximum in the group velocity curve at 4.54 km/sec for a period of 25 seconds which agrees well with the period and velocity of the phase S_* described by Caloi and Gutenberg and the phase S_n of Press and Ewing [1955].

The fundamental oceanic Love mode for case 122 has a secondary maximum of particle amplitude at short periods. For the Canadian shield, Brune and Dorman [1962] have presented a case, CANSD, for which the fundamental Love mode is not associated with channel waves. A secondary maximum does not develop in this case, nor in the Gutenberg-Bullen A model, since the phase velocity does not exceed the lowest channel velocity until the vertical wavelength is greater than the channel thickness. For these models, however, the first shear and the second Love modes are much like the fundamental Love mode for the oceans in that a secondary maximum, which always characterizes these modes, is confined to the low-velocity zone. All these modes are associated with channel waves. The higher modes offer a means of refining the knowledge of the mantle beneath various regions.

The oceanic Rayleigh mode. Oceanic Rayleigh mode phase and group velocities as ob-

served for Pacific paths for periods between 20 and 150 seconds have been presented by *Kuo et al.* [1962]. All the data presented in Figures 27 and 28 of their study, with the exception of those labeled P13 and P14, may be considered representative of the dispersion for Pacific Ocean basins. The paths indicated by data points P13 and P14 cross the Melanesia–New Zealand region of the southwest Pacific and hence will not be included in the present discussion. The scatter of the data below 30 seconds can be attributed to differences in water depth.

The observed phase and group velocities of Kuo et al. were compared with theoretical curves designated 8099S. These curves were obtained by correcting the phase velocities for case 8099 of *Dorman et al.* [1960] according to the spherical correction formula for the Rayleigh wave phase velocities reported by *Bolt and Dorman* [1961] for the Gutenberg-Bullen A continental model.

As was noted in the section on computations,

a program written by L. E. Alsop of the Lamont Geological Observatory was used to check this correction. The exact calculation for case 122 agrees with curves 8099S to within 0.01 km/sec. Thus, the observed oceanic Rayleigh mode phase and group velocities for Pacific paths are in close agreement with those calculated for case 122. In the preceding section it was shown that this case agrees with dispersion data for oceanic Love waves. These comparisons indicate that case 122 must not depart greatly from the conditions to be found in the mantle beneath the oceans, since it satisfies both Love and Rayleigh wave data.

Rayleigh wave amplitudes for case 122, comparable with the amplitudes for Love waves in Figure 3, are shown in Figure 4. The distribution of shear velocity with depth is again given at the left side of the figure. The vertical displacements are normalized to unity at the water surface. Particle amplitudes for case 122 are shown for one calculation for a spherical earth and three
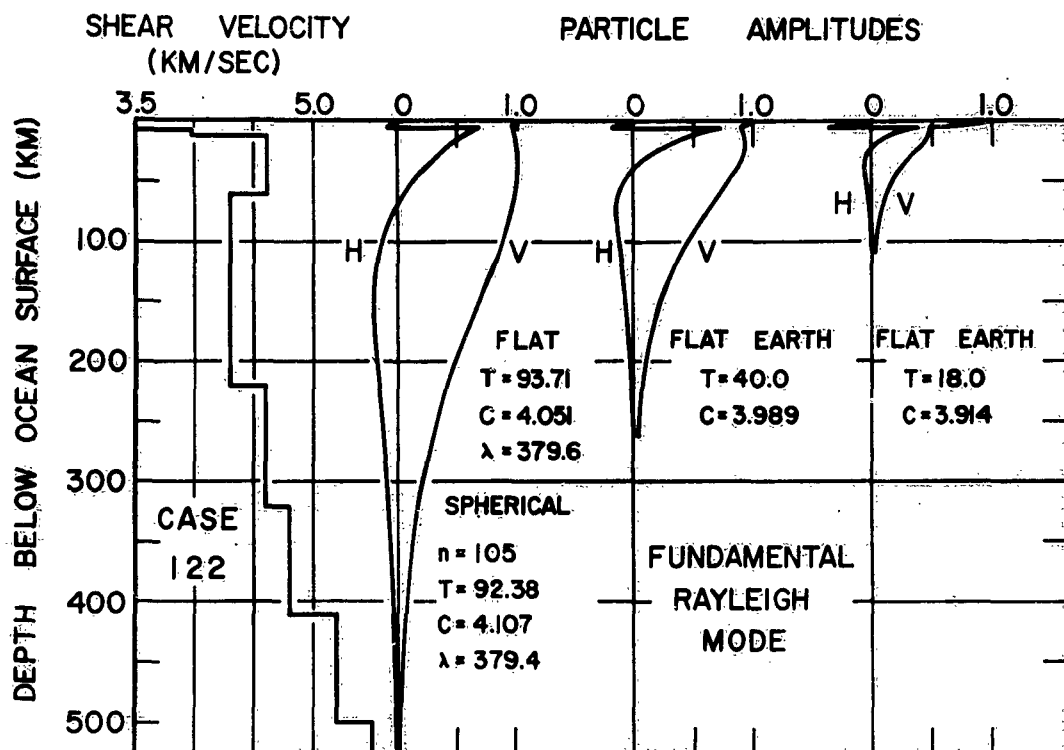

Fig. 4. Particle amplitude profiles as a function of depth for the Rayleigh mode of case 122. Vertical displacement V, normalized to unity at the free surface. H is horizontal displacement. H and V of same sign indicate retrograde motion. For a given wavelength, λ, particle amplitudes are the same for calculations for flat and spherical models. To first order, the group velocities are also the same.

calculations for flat layers. No secondary maximum or channel wave develops in the low-velocity channel for the oceanic Rayleigh mode of case 122.

When calculations of Rayleigh mode particle amplitudes for spherical and flat-layered versions of case 122 are compared for a given wavelength (Figure 4), it is found that the particle amplitudes are nearly the same. For periods shorter than 100 seconds, the group velocities agree to within 0.02 km/sec. *Bolt and Dorman* [1961] also reported that the Rayleigh mode group velocities calculated for spherical and flat-layered versions of their models are nearly the same, even though the phase velocities are different. To extend the discussion of the relation between phase and group velocities as a function of wavelength for a spherical and a flat-layered medium, let us consider one of the well-known expressions which relates the group velocity U to the phase velocity C for a certain wavelength λ:

$$U_s = C_s - \lambda \, dC_s/d\lambda \qquad (1a)$$

where the subscript s indicates a spherical earth. Similarly, for a flat earth,

$$U_f = C_f - \lambda \, dC_f/d\lambda \qquad (1b)$$

If the group velocities and wavelengths are approximately equal, we may subtract (1b) from (1a) and obtain

$$0 \approx (C_s - C_f) - \lambda[dC_s/d\lambda - dC_f/d\lambda] \qquad (2)$$

This may be rearranged as

$$(C_s - C_f)/\lambda \approx d(C_s - C_f)/d\lambda \approx m \qquad (3)$$

where m is a constant. Alternatively, we may write

$$C_s \approx C_f[1 + T \, d(C_s - C_f)/d\lambda] \\ \approx C_f[1 + mT] \qquad (4)$$

The last of equations 4 is identical to the empirical formula (12) of *Bolt and Dorman* [1961]. These authors found that $m = 0.00016$ for the Gutenberg-Bullen A *continental* model.

The curves 8099S of Kuo et al. resulted from the application of the Bolt and Dorman sphericity correction to the Rayleigh mode phase velocities for Dorman's *oceanic* case 8099. As was previously stated, oceanic phase and group ve-

locities for case 122 have been calculated using a program for Rayleigh waves on a spherical earth. These calculations agree with the results presented by Kuo et al. as curves 8099S. Thus the correction for the effect of sphericity on Rayleigh waves is nearly identical for these geologically reasonable models, oceanic and continental.

In contrast to the correction for sphericity for Rayleigh waves, which is the same for many models, the corrections for sphericity for fundamental and higher-mode Love waves and for the shear modes of the Rayleigh type are quite dependent on the choice of physical parameters in the mantle. Calculations for cases 122 for oceans, CANSD for the Canadian shield, and the Gutenberg-Bullen A model show quite different sphericity corrections for the fundamental Love mode. The fundamental Love mode for the oceanic model develops a channel wave, the short-period components of the G wave; the fundamental Love mode has no channel wave for these continental models.

Future problems. Several problems remain to be solved for the mantle beneath the oceans. For example, the calculated phase velocities of oceanic Rayleigh waves of *Kuo et al.* [1962] are a few hundredths of a kilometer per second high for periods greater than 80 seconds. Similarly, the calculated phase velocities for the Gutenberg-Bullen A model [*Bolt and Dorman*, 1961] are slightly above the data for periods greater than 150 seconds.

In addition, the calculated Love wave phase velocities in Figure 1 of this paper (spherical model) are also several hundredths of a kilometer per second too high for periods greater than 150 seconds. As a consequence, the calculated group velocities are somewhat low for periods between 100 and 250 seconds. Thus both Rayleigh and Love waves indicate that small changes in either velocity or density must be made in the theoretical models. Preliminary investigations show that either shear velocities or densities must be altered at depths in excess of 200 km.

Between 10 and 20 seconds, the calculated Love wave group velocities in Figure 2 lie above much of the data. Adjustments of shear velocity in the crust and sediments may remove part of this discrepancy. Changes may also be needed in the shape of the low-velocity channel. These

adjustments will probably somewhat alter the shape of the particle amplitude curves for the fundamental Love mode.

Observational data for higher modes were not considered in the present study. These modes should be observed and the observations explained by the model used to fit the fundamental modes. Finally, we note that S wave data, still unavailable for oceanic areas at epicentral distances between 1° and 17°, and between 17° and 34°, would be of great help in future studies.

Summary and conclusions. 1. Love wave dispersion for a spherical earth has been computed for a model of the suboceanic crust and mantle, case 122. This model, which is similar to case 8099 of *Dorman et al.* [1960], has a low-velocity channel.

2. Theoretical Love wave velocities for case 122 explain phase and group velocity data for oceanic Love waves over a broad range of periods to within a few hundredths of a kilometer per second.

3. An oceanic Rayleigh wave program for a spherical earth, written by L. E. Alsop, has been used to confirm the spherically corrected phase and group velocities for Dorman's case 8099 (presented by *Kuo et al.* [1960] as case 8099S). These theoretical results explain the oceanic Rayleigh wave data of Kuo et al. to within a few hundredths of a kilometer per second. The Love and Rayleigh wave results can be further improved by minor modifications of the model.

4. The effect of sphericity on Love waves is found to be of great importance and to be dependent on the model under consideration. For the oceans, the differences between the phase velocities which result from calculations for flat-layered and spherical earth models extend to periods as short as 10 seconds.

5. Thus, since a single isotropic distribution accounts for oceanic Love and Rayleigh wave data, the discrepancy between calculations for flat layers is resolved by sphericity without the need for a difference in SH and SV velocities in the mantle beneath the oceans.

6. *Brune and Dorman* [1962] found, for the mantle beneath the Canadian shield; (1) that a low-velocity zone is required and (2) that no difference between SH and SV velocities is needed. Thus their study and this paper show that these two conclusions are applicable to large areas of the world.

7. For periods shorter than 60 seconds, the G wave, which is the fundamental oceanic Love mode in this range of periods, develops a large secondary maximum of particle amplitude in the low-velocity channel and may be considered a channel wave. For periods between 60 and 300 seconds, the free surface and the steep gradient in shear velocity down to approximately 900 km form the wave guide of the G wave. At longer periods the particle motion penetrates the high-speed lower mantle, causing phase and group velocities to rise sharply.

8. Calculated higher-mode Love waves and the shear modes of Rayleigh type also exhibit secondary maximums in the low-velocity channel when their phase velocities are slightly greater than the channel velocity and their vertical wavelengths are less than the channel thickness. These secondary maximums will be important in the consideration of problems of excitation and source depth.

9. It is useful to compare the mantle velocities determined from surface wave data for oceanic and continental areas. Examples include case 122, case 8099 of *Dorman et al.* [1960], and case 6EGHP1 of *Aki and Press* [1961] for the oceans and, for the continents, the Canadian shield model, CANSD, of *Brune and Dorman* [1962] and the Gutenberg-Bullen A model discussed in studies of the Chilean earthquake. When this comparison is made, it is confirmed that channel velocities are smaller beneath the oceans. Further study is needed to determine whether these channel velocities usually occur at shallower depths beneath ocean basins than beneath continents.

Vela program through the Air Force Cambridge Research Center under contract AF 19 (604) 7376.

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